Photon strength in heavy nuclei
in correlation to the
number of valence nucleons

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1. *Broken spherical and axial symmetry*
2. *Giant resonances and e.m.radiation*
3. *Rotation invariants*
4. *Electric diple strength in the GDR tail*
5. *Valence nucleon systematics*

work in progress

Budapest, September 2009

EFNUDAT workshop

***The deformation of nuclei** (i.e. a non - spherical shape) has a strong impact on many nuclear problems – but its origin and parameterization still need further study*

on the

1. ***fission process***
2. ***deacy of fission products***
3. ***interpretation of n - capture data, especially when γ-decay is involved.***

through

1. ***measurements like Coulomb excitation***
2. ***'macroscopic' calculations (Bohr Hamiltonian)***
3. ***self consistent 'microscopic calculations' (shell model, random phase approximation)***

etc.

Parameterizations and models using a Lorentzian fit to the isovector giant dipole resonance (GDR) are strongly hampered by the deformation induced widening of the GDR mocking up a larger width.

*In the past this was accounted for by using **two Lorentzians** for strongly deformed nuclei, but in nuclei with small and / or triaxial deformation this effect has been identified only recently.*

*The present experimental wisdom on nuclear **deformation and triaxiality** will be reviewed to demonstrate the impact on neutron capture physics – especially the **photon strength function** (PSF).*

A. R. Junghans et al., Physics Letters B 670, 200 (2008); E. Grosse et al., contribution to CGS-13.

Triaxiality in Nilsson-Strutinski calculations (FRDM-HFB)

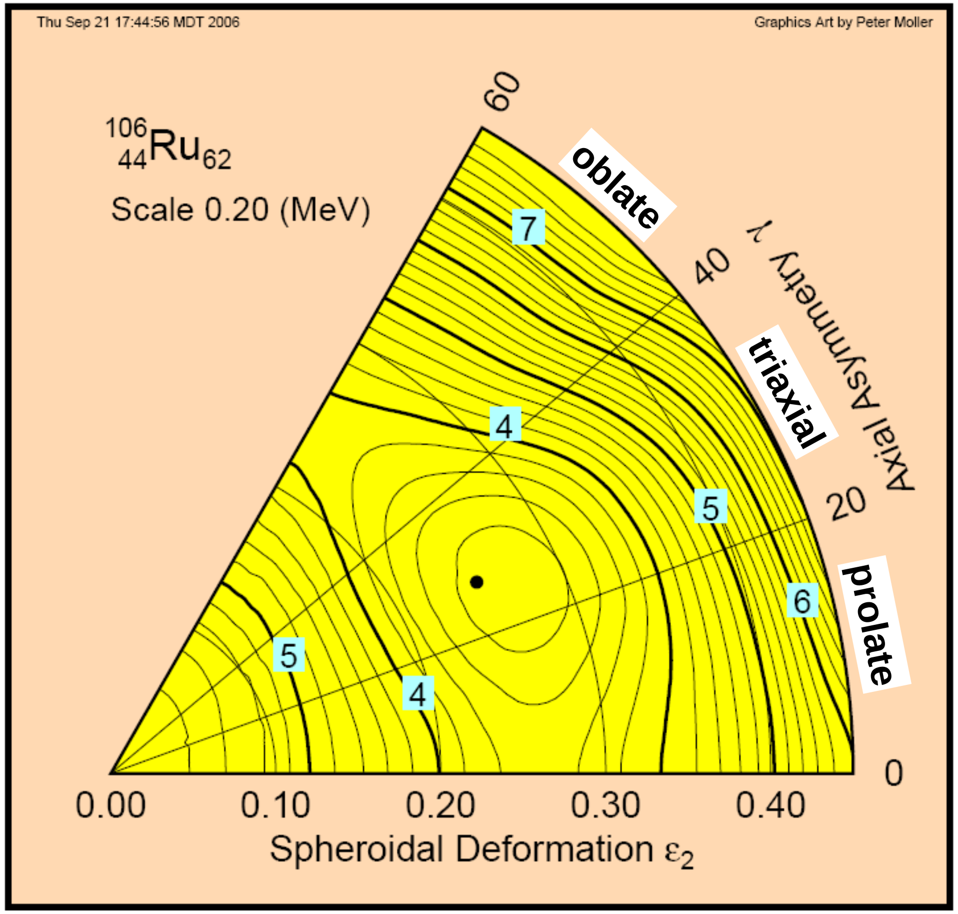


Fig. 4. The calculated ground state shape of ^{106}Ru is triaxial, as is the case for several hundred other nuclei across the nuclear chart, out of ~ 9000 studied.

and in calculations with the Thomas-Fermi plus Strutinsky integral (ETFSI) method, saying:

*We are thus inclined to accept the widespread (>30%) occurrence of **triaxiality**…as being an essential feature of ETFSI calculations, if not of the **real world** …albeit the associated reduction in energy, …never exceeds 0.7MeV.*

P. Möller et al., PRL 97(06) 162502

A. K. Dutta et al., PRC 61 054303

Giant resonances

and their e.m. excitation

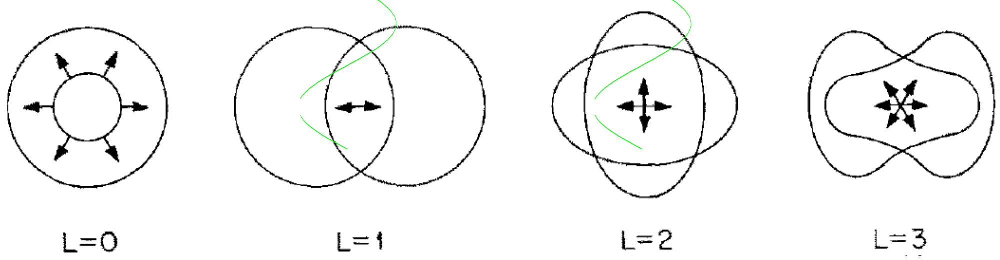
seen as

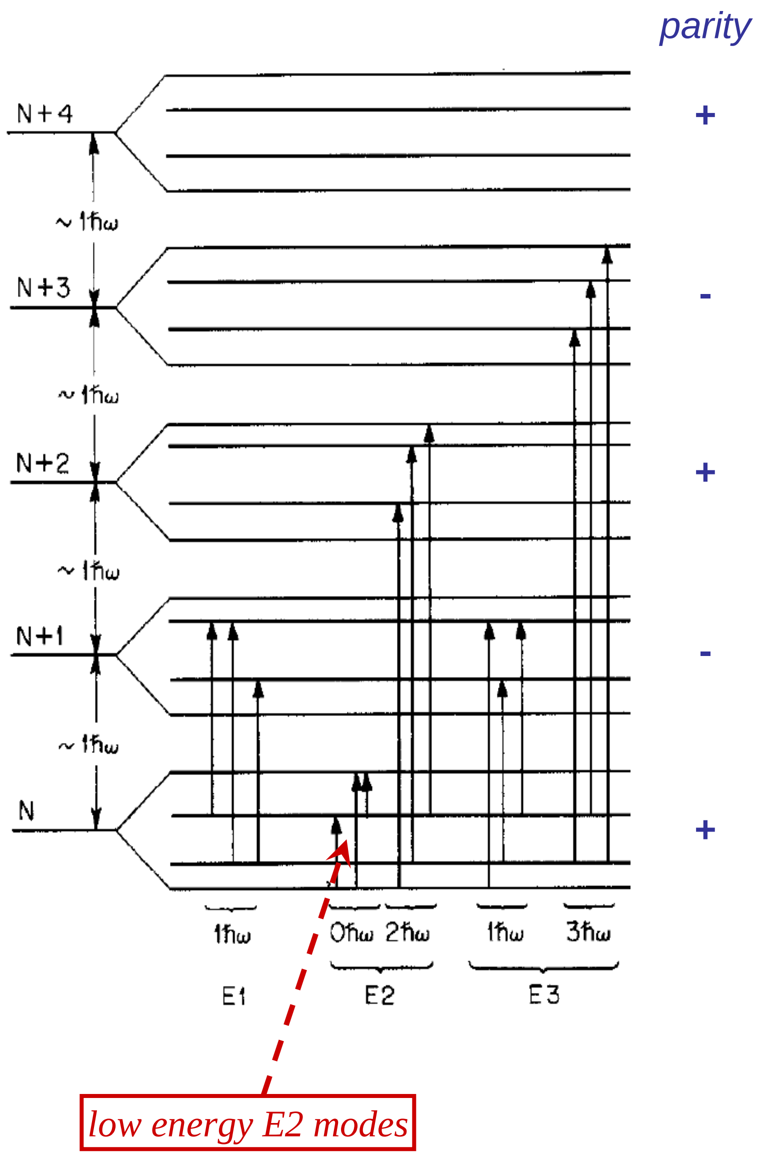
particle-hole configurations

or as

collective modes of

the droplet of nuclear matter





ORNL, F.Bertrand, NPA354 (1981) 129c

Electromagnetic transitions reveal collective modes of nuclei ;

rotation invariants *should be used for a model-free interpretation, as they are* ***observables*** *with the same value in the body-fixed frame as in the laboratory.*

For E2 transitions from the ground state 0 one can form quadrupole tensor products coupled to ***angular momentum 0*** *– i.e. rotation invariant quantities:*

$$K_2 = |\langle 0| [\mathrm{E2} \times \mathrm{E2}]_0 |0\rangle|^2 = \sum_{r=1,\infty} \langle 0\|\mathrm{E2}\|\, 2_r\rangle\langle 2_r\|\mathrm{E2}\|0\rangle = \sum_{r=1,\infty} |\langle 2_r\|\mathrm{E2}\|0\rangle|^2 \equiv \frac{5}{16\pi} Q^2_{rms} = (\frac{3}{4\pi} Z R_0^2 d)^2$$

K_2 *is an invariant deformation and* ***K_3*** *determines a rotation invariant triaxiality:*

$$K_3 = \sqrt{\frac{7}{10 K_2^3}} \langle 0 | \{[\mathrm{E}\,2 \times \mathrm{E}2]_2 \times \mathrm{E}2\}_0 \,|\, 0\rangle = \sqrt{\frac{7}{10 K_2^3}} \sum_{r,s=1,\infty} \langle 0\|\mathrm{E2}\|2_r\rangle\langle 2_r\|\mathrm{E2}\|2_s\rangle\langle 2_s\|\mathrm{E2}\|0\rangle$$

$$= -\cos\,(3\delta)$$

$$K_4 = \frac{\langle 0 \,|\, \{[\mathrm{E}\,2 \times \mathrm{E}2] \times [\mathrm{E}2 \times \mathrm{E}2]\ \}_0 \,|\, 0\rangle}{K_2^2}$$

Assuming ***reflection symmetry*** *and* ***equal distribution of charge*** *one gets with* ***<u>no</u>*** *other assumptions:*

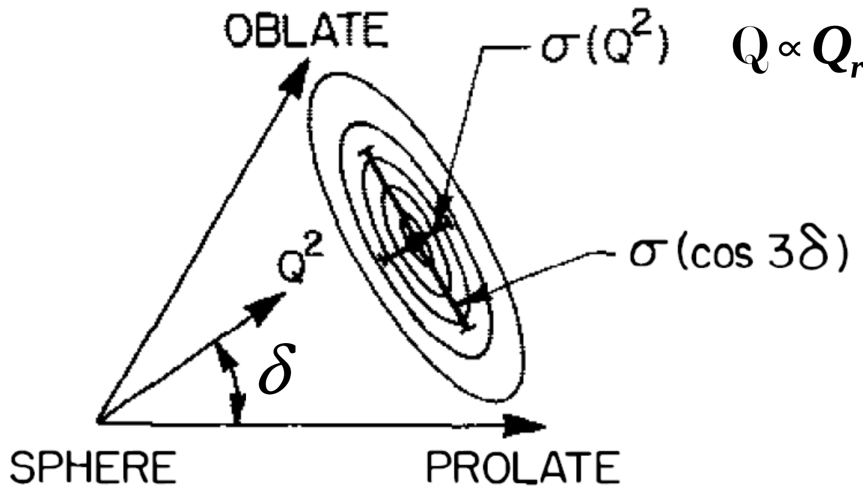
$$Q_{rms}\ \cos\,\delta = \frac{Z\,(2R_3^2 - R_1^2 - R_2^2)}{5} \qquad R_0^3 = R_1 R_2 R_3 = (1.2)^3 A$$

$$Q_{rms}\ \sin\,\delta = \frac{\sqrt{3}\, Z\,(R_1^2 - R_2^2)}{5} \qquad \langle r^2\rangle = \frac{R_1^2 + R_2^2 + R_3^2}{5}$$

δ, R_i *are rms values for the departure from axial symmetry and for the radii of the triaxial body.*

K. Kumar, Phys. Rev. Lett. **28**, 249 (1972)   D. Cline, Annu. Rev. Nucl. Part. Sci. 36 (1986) 683   V. Werner et al., PRC **71**, 054314 (2005)

The ***variances*** *of the rms quantities can be deduced from the higher order invariants:*



$Q \propto Q_{rms}$

Q^2:
- Centroid $\langle S | Q^2 | S \rangle$
- Width $\sigma(Q^2) = \sqrt{\langle Q^4 \rangle - (\langle Q^2 \rangle)^2} \quad \propto K_4$

$\cos 3\delta$:
- Centroid $\langle S | Q^3 \cos 3\delta | S \rangle$
- Width $\sigma(\cos 3\delta) = \sqrt{\frac{\langle Q^6 \cos^2 3\delta \rangle}{\langle Q^6 \rangle} - \left(\frac{\langle Q^3 \cos 3\delta \rangle}{\langle Q^3 \rangle}\right)^2} \quad \propto K_5$

Figure 4 Distribution plot of the parameters Q^2 and δ required to define the E2 properties in the intrinsic frame. All possible E2 moments are defined by the region $Q \geq 0$ and $0° \leq \delta \leq 60°$.

D. Cline, Annu. Rev. Nucl. Part. Sci. 36 (1986) 683

Variances of rms quantities

C.Y. Wu et al. / Nuclear Physics A 607 (1996) 178

$$Q^2 = K_2 = \frac{5}{16\pi} Q_{rms}^2$$

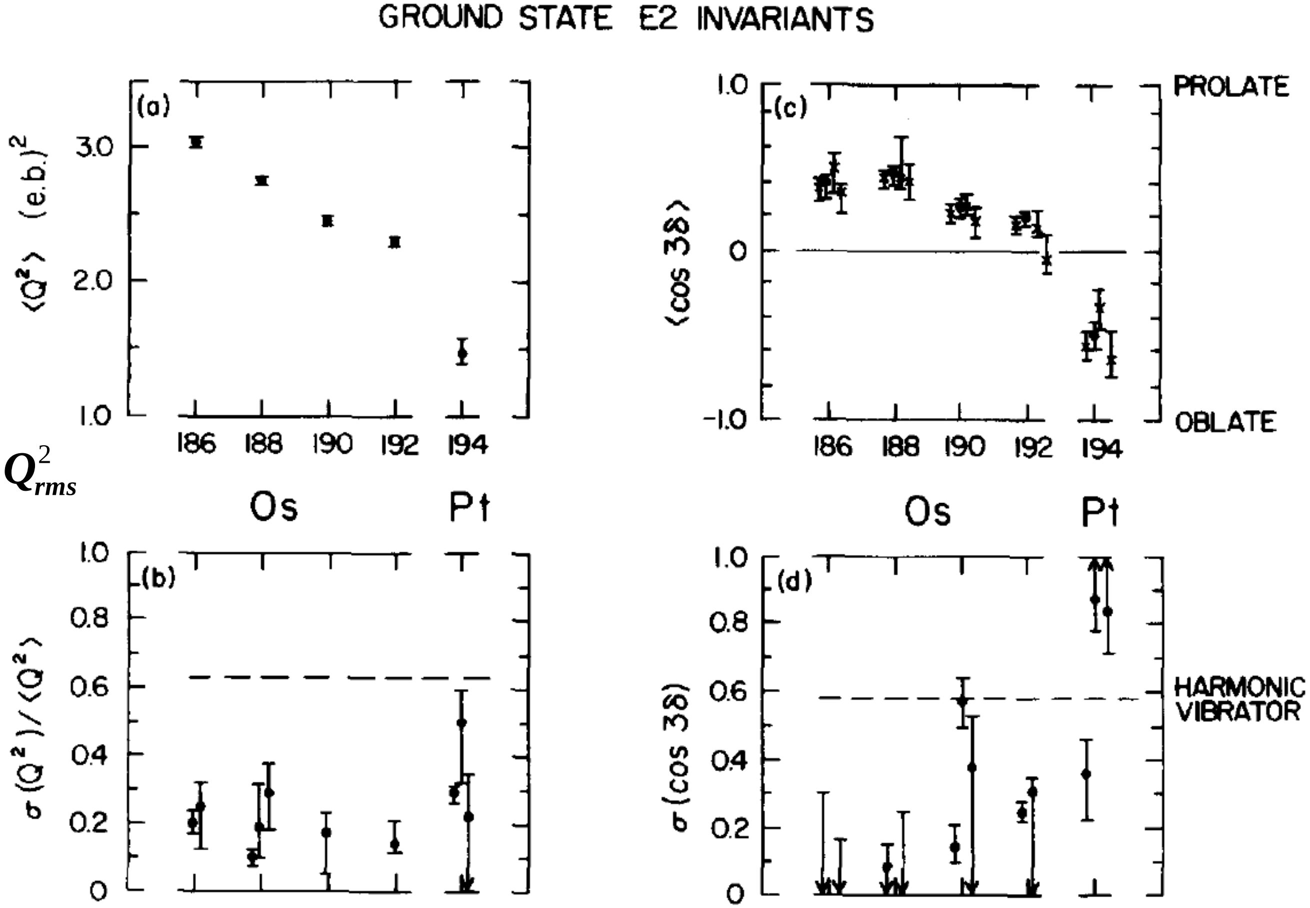
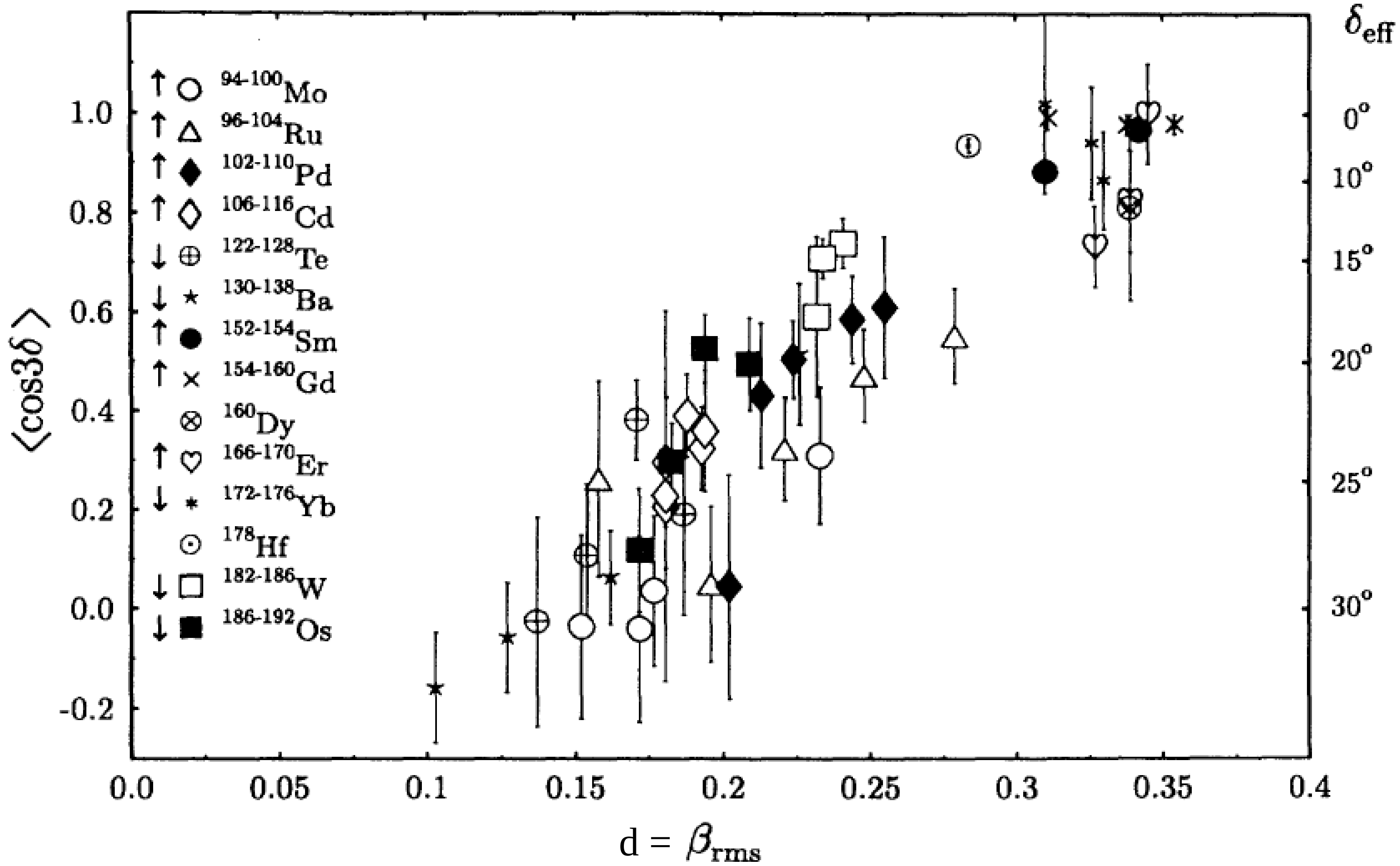


Fig. 25. Centroids (a,c) and vibrational widths (b,d) for the magnitude and asymmetry of quadrupole deformation of the ground state in ^{186,188,190,192}Os and ^{194}Pt. Three independent sum rules for determining $\sigma(Q^2)$ and six independent sum rules for determining $\sigma(\cos 3\delta)$ were used to check the completeness of the summations. The extracted vibrational widths are not shown if they are smaller than zero.

Correlation of deformation and triaxiality



W. Andrejtscheff and P. Petkov, PRC 48 (93) 2531

Without any new parameter

σ(γ,n) is well described

for all Nd-isotopes

using :

E_0 from FRDM,

Γ from hydrodynamics,

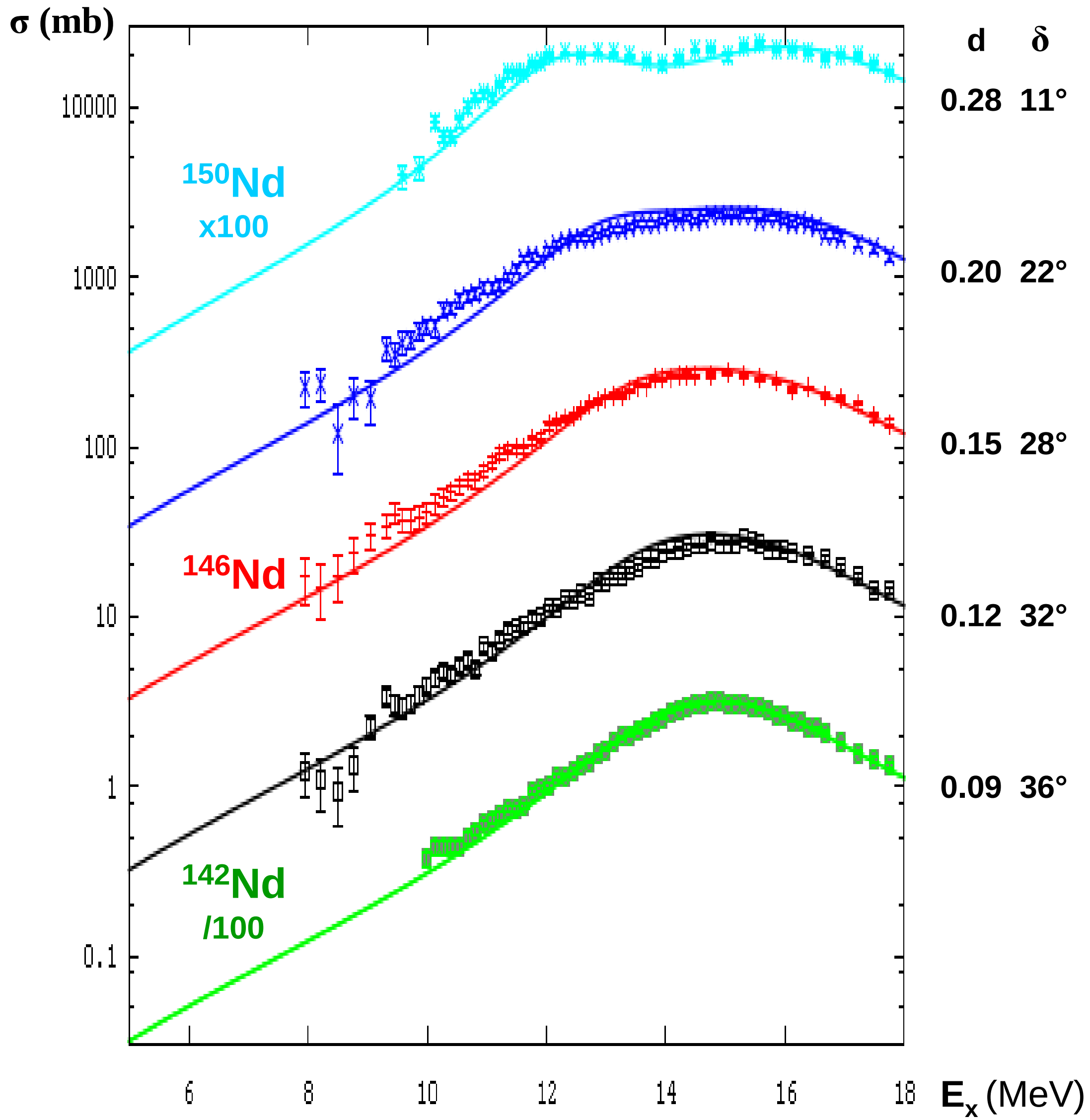
strength from TRK sum rule

and triaxiality from systematics.

$$\sigma_\gamma = \sum_{k=1,2,3} \frac{2I_j}{\pi} \frac{E^2 \Gamma_k}{\left(E_k^2 - E^2\right)^2 + E^2 \Gamma_k^{\ 2}};$$

$$E_k = E_0 \exp\left\{-\sqrt{5/4\pi}\, d \cos(\delta - \tfrac{2}{3} k\pi)\right\};$$

$$\Gamma_k = 0.050 (E_k)^{1.6}$$

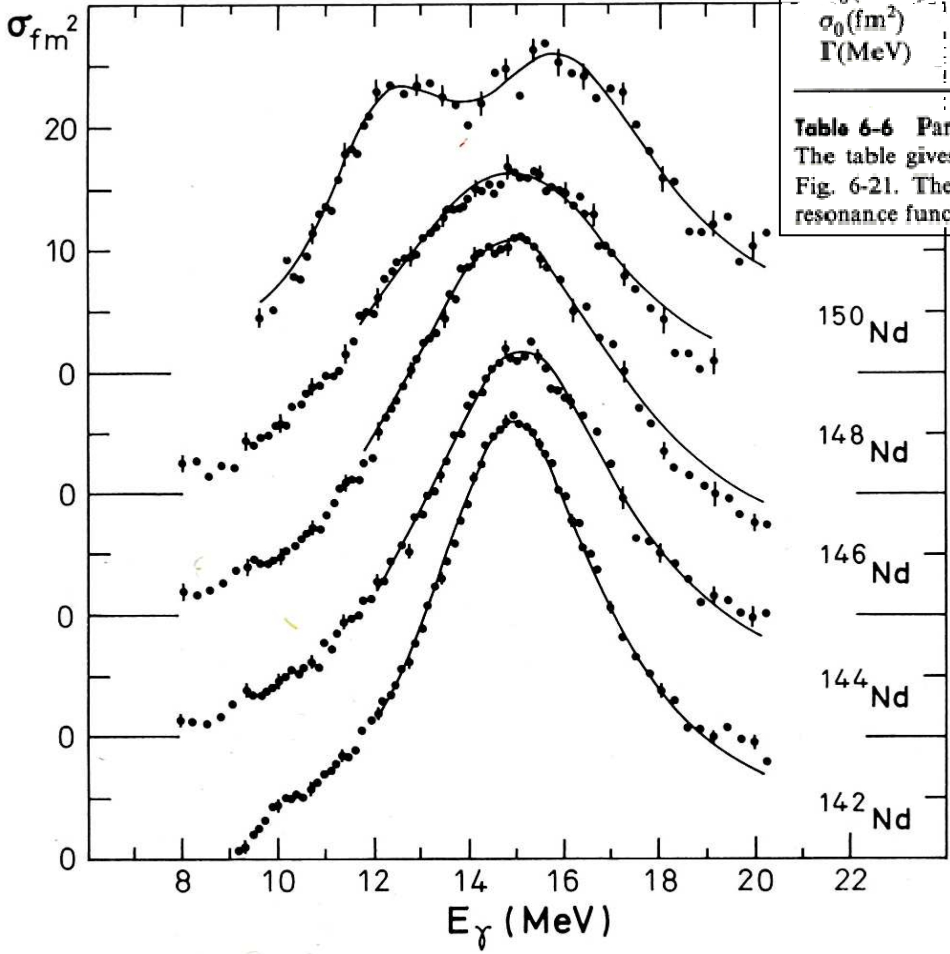


A. R. Junghans et al., Physics Letters B 670, 200 (2008)

data from: P. Carlos et al., NP A172 (1971) 437

Deformation induced GDR splitting –

known since long, but interpreted differently



	^{142}Nd	^{144}Nd	^{146}Nd	^{148}Nd	^{150}Nd	
E_0(MeV)	14.9	15.0	14.8	14.7	12.3	16
σ_0(fm^2)	36	32	31	26	17	22
Γ(MeV)	4.4	5.3	6	7.2	3.3	5.2

Table 6-6 Parameters for the dipole resonance in even neodymium isotopes. The table gives the parameters for the Lorentzian resonance curves drawn in Fig. 6-21. The cross section for ^{150}Nd has been fitted to the sum of two resonance functions.

if weakly deformed nuclei are treated as spherical the <u>apparent</u> width is enlarged

Lorentzian fits seem to show increase of width with A

σ_0 seems to fluctuate with A

Figure 6-21 Photoabsorption cross section for even isotopes of neodymium. The experimental data are from P. Carlos, H. Beil, R. Bergère, A. Lepretre, and A. Veyssière, *Nuclear Phys.* **A172**, 437 (1971). The solid curves represent Lorentzian fits with the parameters given

Bohr & Mottelson, Nuclear Structure, Vol. II

Adiabatic coupling of E1 to E2 modes

The dipole vibrations in the GDR are much faster than the quadrupole vibrations. Thus an adiabatic approximation for the GDR-splitting can be applied and then the pole energies of the GDR components are given by:

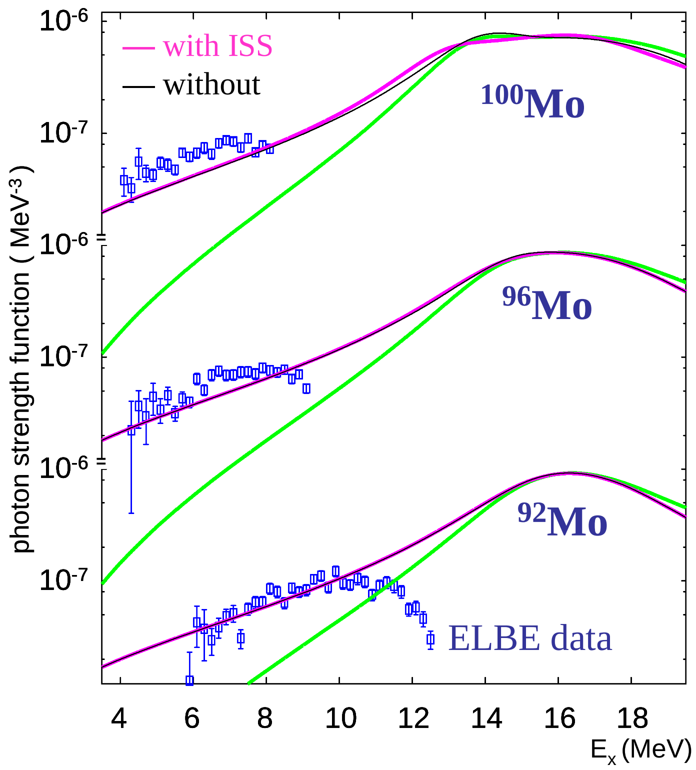
$$E_k = \hbar\omega_k = \frac{E_0 R_0}{R_k}$$

The Hill & Wheeler relation also holds for the rms quantities:

$$R_k = R_0 \exp\left\{\sqrt{5/4\pi}\, d \cos(\delta - \tfrac{2}{3}k\pi)\right\}$$

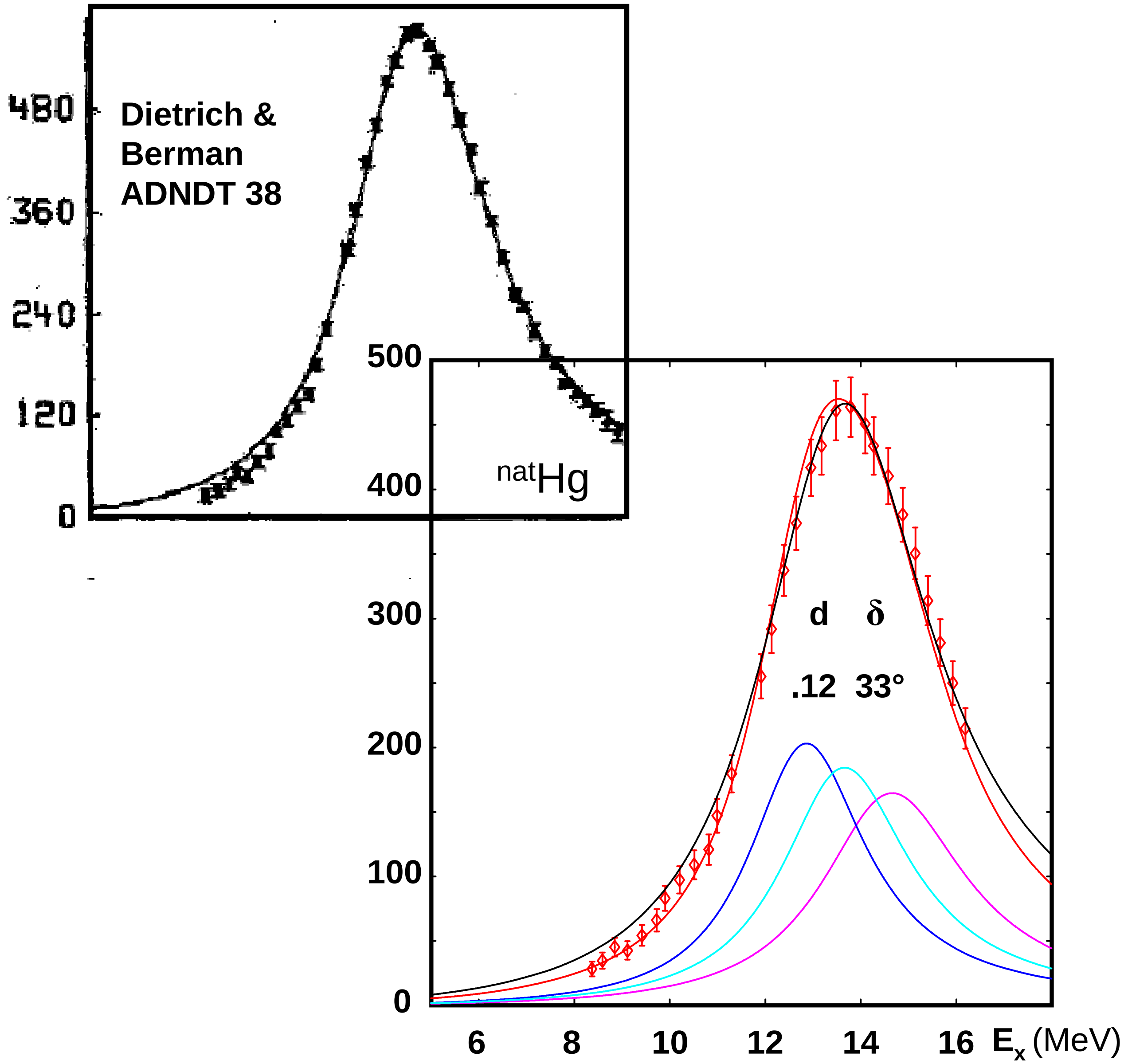
$$R_1 R_2 R_3 = R_0^3.$$

The adiabaticity was tested by instantaneous shape sampling (ISS) applied to the GDR-Lorentzians for ^{92,96,100}Mo (using IBA-parameters)



S.Q. Zhang et al., PRC 80 (2009) 021307; M. Erhard et al., subm. to PRC

Impact of deformation and triaxiality on photon strength near threshold.



For 'soft triaxial' Hg nuclei a fit with <u>one</u> Lorentzian results in Γ_{eff} ~ **4.4 MeV.**

We get a perfect agreement to 3 superimposed Lorentzians with $\Gamma_{1,2,3}$ ~ **3.2 MeV** ***(in red).***

The low energy tail is strongly influenced by this difference, as depicted ***in black****.*

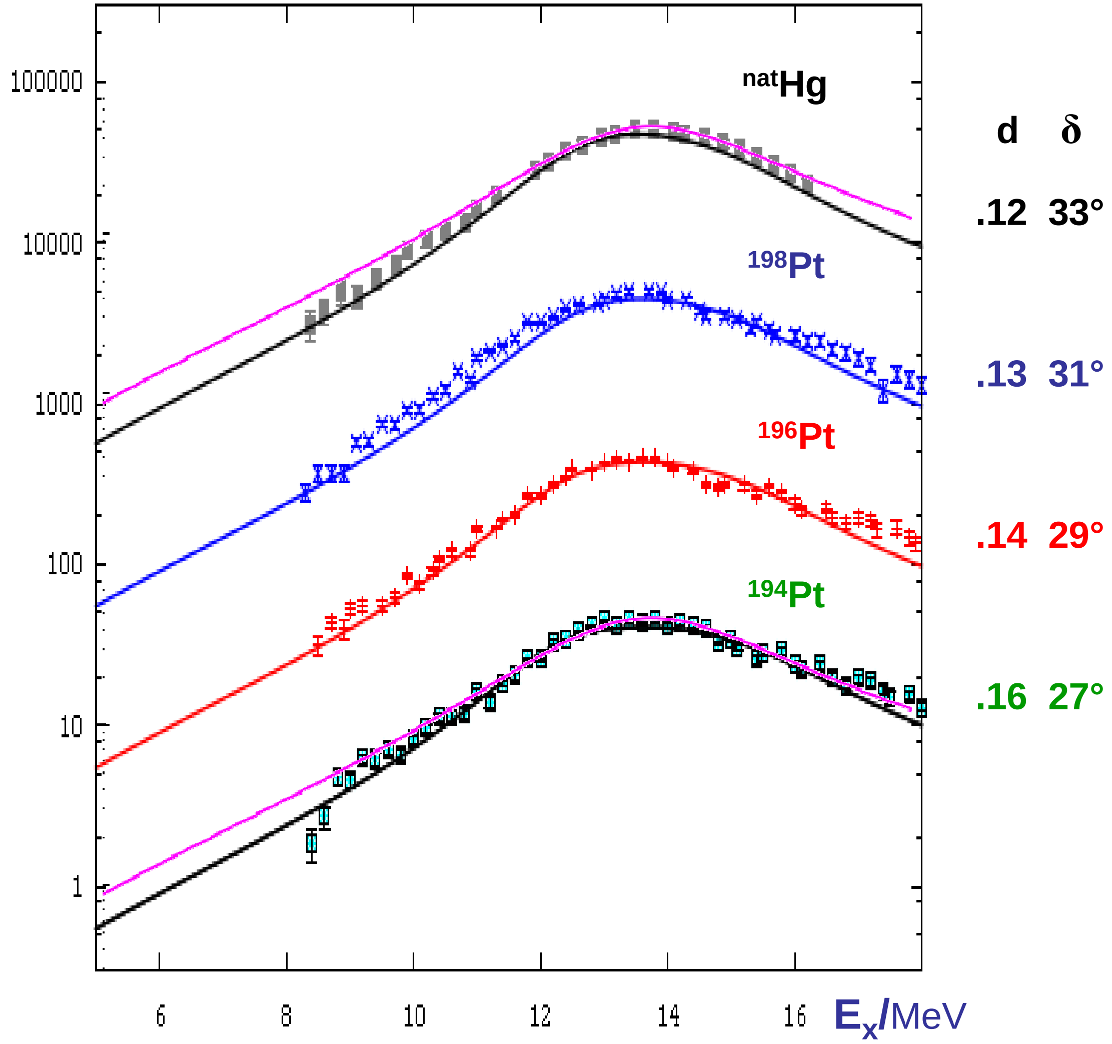
data from: Veyssiere, CEA Saclay

Impact of deformation and triaxiality on photon strength near threshold.

For 'soft triaxial' nuclei ^{194}Pt, ^{196}Pt, ^{198}Pt and Hg

a fit with one Lorentzian (pink) results in $\Gamma_{eff} \sim 4$ - 5 MeV, and in too high yield at low E_x.

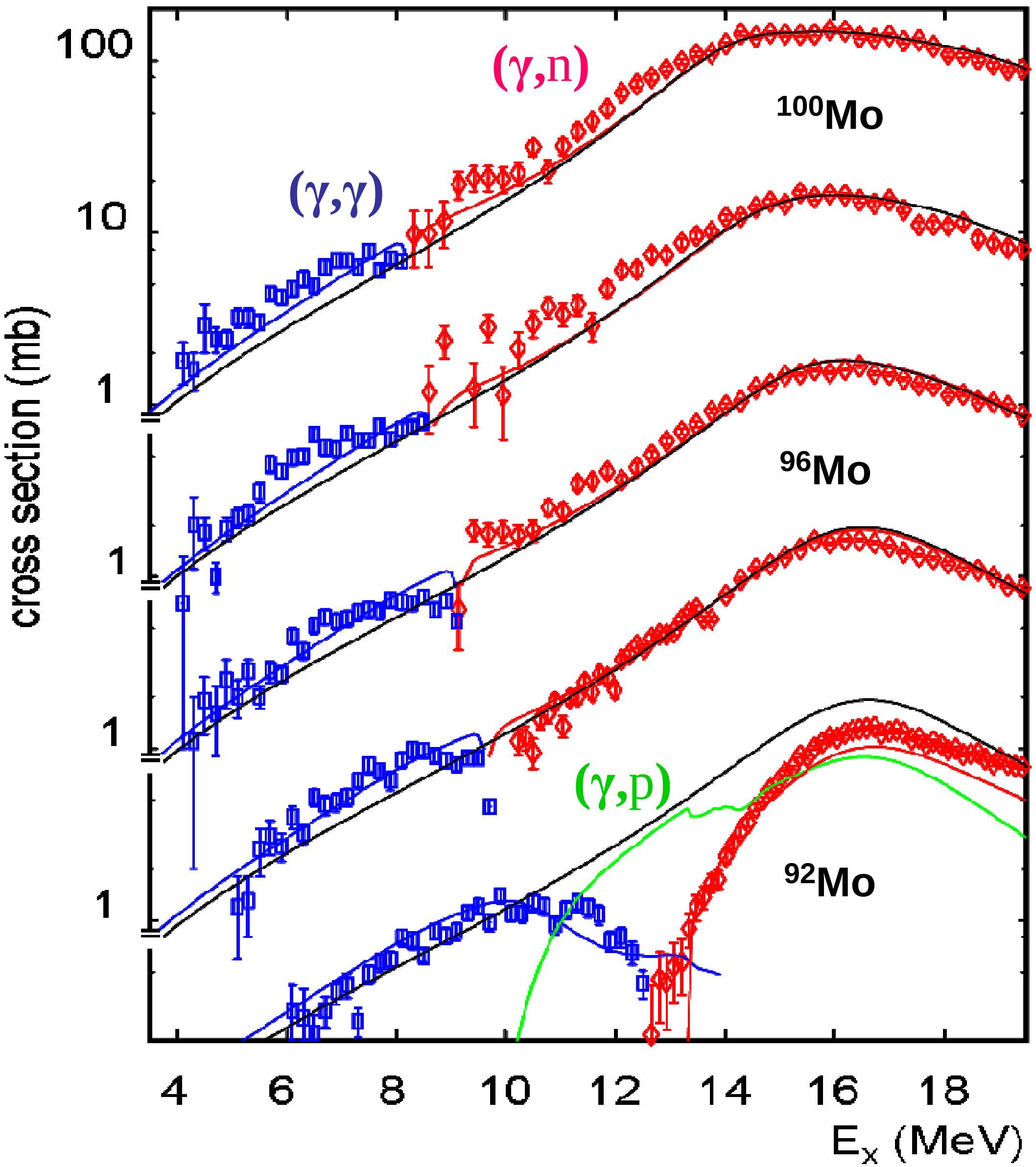
We get a perfect agreement to 3 superimposed Lorentzians with $\Gamma_{1,2,3} \sim 3.2$ MeV.



data from: CEA Saclay

Scattering **(γ,γ)**
and (γ,n) ***–data***
match perfectly
for Mo-isotopes.

Photon strength function as input for Hauser Feshbach calculations yields good description for $5\ MeV < E_x < 20\ MeV$.

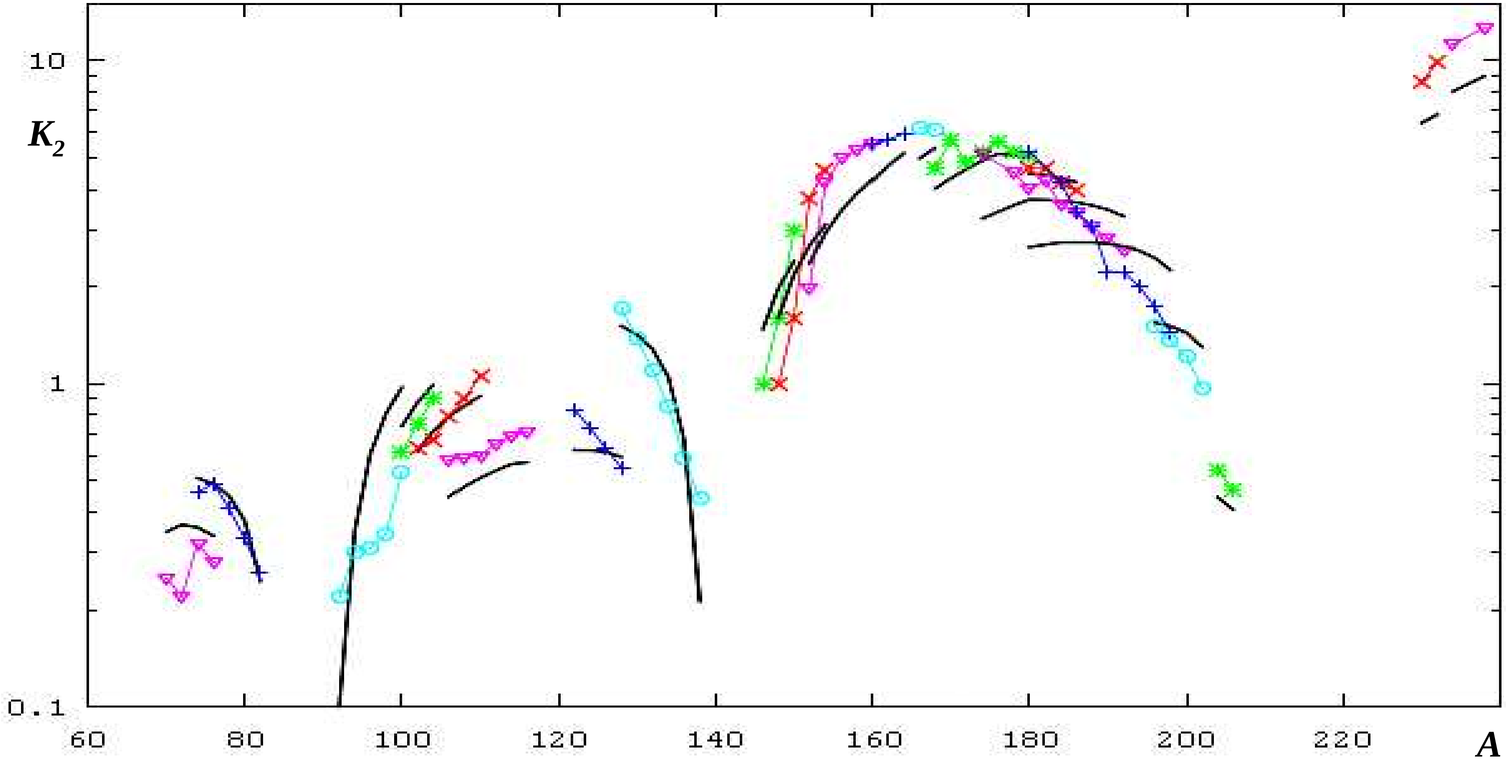


(γ,γ): ELBE, M.Erhard et al., PRC, to be published.

Calculations with TALYS, A.Koning et al.

(γ,n) –data from CEA Saclay

Attempt to find a parameterization for K_2 in terms of Z and A.



$$\frac{V_p V_n Z N}{V_p + V_n}$$

V_p : *number of valence protons etc.*

By appropiate normalization the correlation between $\boldsymbol{K_2}$ *(and thus the rms deformation* d*) and the number of valence protons and neutrons can be extended to the full range of heavy nuclei 70 < A < 238.*

Conclusions

- ***Rotation invariant observables*** *(defined by quadrupole moments and transitions)* ***allow to quantify the braking of spherical and axial symmetry in nuclei without a theoretical model***
- ***Such information has been determined for ~ 100 nuclei with A>70***

  – more than 30 % are triaxial
- ***An unexpected ccorrelation is observed between quadrupolar deformation and triaxiality***
- ***The rms axis lengths of the intrinsic ellipsoid can be directly deduced from the invariants***
- ***The corresponding oscillator frequencies determine a splitting of the GDR into 3 components*** *(at E_k)*
- ***Each component is widened by spreading, which is proportional to $E_k^{1.6}$***
- ***Deformation and triaxiality depend on the number of valence nucleons***